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A Unified Framework for Social Graph Modeling and Node Decision-Making via Differentiable Mixed-Integer Optimization

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Abstract: This paper proposes a differentiable mixed-integer optimization-driven approach for social graph modeling, targeting the widespread discrete structural decision problems in social data. The method integrates a mixed-integer programming module into a graph neural network, constructing a structure selection mechanism with integer constraints to jointly model tasks such as social node selection and information propagation path construction. During optimization, Lagrangian relaxation and KKT approximation are introduced to enable gradient propagation in the originally non-differentiable integer decision process, supporting end-to-end structure-aware training. To enhance the model's expressive power for complex graph structures, a structural regularization term is incorporated to align the node selection process with the graph topology. The experimental section includes a series of sensitivity analyses, examining the effects of regularization strength, graph density, and relaxation parameters on propagation performance. The results show that the proposed method improves information diffusion and structural modeling quality without increasing parameter count, demonstrating strong practicality and generalization ability. This approach provides a new optimization perspective for structural modeling in social data, enabling high-quality structural reasoning and node decision-making in complex social environments.

Keywords: Structure-aware modeling; Differentiable optimization; Discrete decision making; Social network communication

1. introduction

With the widespread adoption of social media, the volume of information embedded in social data is growing exponentially. This data encompasses multiple dimensions, including user behavior, interest preferences, social structures, and public opinion dynamics. The abundance of social data presents unprecedented opportunities for modeling human social behavior[1]. It also provides a solid foundation for key tasks such as personalized recommendation, opinion diffusion, community detection, and misinformation identification. However, social data is inherently high-dimensional, heterogeneous, and fast-evolving. It also involves discrete decision variables. These complex characteristics pose significant challenges to traditional optimization methods when modeling and solving such problems[2]. Therefore, a new modeling and optimization paradigm is urgently needed. This paradigm must be capable of capturing both continuous relationships and discrete structural decisions to fully uncover the latent knowledge structures within social data[3].

Mixed-Integer Optimization (MIO) combines continuous and integer variables and offers a theoretically sound tool for modeling combinatorial structures in social data. For example, tasks such as influence maximization, user clustering, and graph structure learning often require discrete selection over nodes or edges in social networks. These are essentially combinatorial optimization problems with integer constraints. However, traditional MIO methods rely heavily on heuristic search or branch-and-bound strategies[4]. They are computationally expensive and difficult to integrate into modern deep learning frameworks. In practical social data mining applications, their non-differentiability severely limits their ability to collaborate with deep neural networks. This also makes end-to-end training and inference difficult to implement[5].

To address these issues, Differentiable Mixed-Integer Optimization (DMIO) has gained increasing attention in recent years. This direction aims to bridge the gap between optimization theory and differentiable computation. It transforms MIO problems into smooth approximations or employs strategies such as Lagrangian relaxation, KKT conditions, or dualization to enable gradient propagation through integer-constrained problems[6]. As a result, DMIO retains the expressive power of discrete modeling while integrating naturally into neural networks. It allows models to learn optimal decisions directly from data when handling complex social structures. In areas such as graph neural networks, reinforcement learning, and self-supervised modeling, DMIO provides a unified optimization framework that offers new possibilities for solving structural problems in social data[7].

Many key tasks in social data involve structured reasoning and decision-making. These include selecting optimal partitions in community detection, activating seed nodes in influence propagation, and constructing discrete preference paths in social recommendation. These problems require both continuous modeling capacity and discrete decision capability. Traditional neural networks struggle to express such structured constraints. They often rely on relaxed approximations or post-processing steps, leading to unstable optimization results and poor generalization. Introducing DMIO enhances the model's ability to accurately capture structural information. It also ensures both feasibility and optimality within an end-to-end learning process. This allows the model to better reflect the discrete and complex nature of real-world social behavior.

Driven by advances in differentiable optimization, social data mining is undergoing a paradigm shift. Traditional approaches, which focus on probabilistic modeling and continuous function optimization, are evolving toward unified frameworks that integrate structural optimization, differentiable integer modeling, and neural representation learning. This evolution not only expands the expressive power of models but also promotes a more mechanistic understanding of complex social systems. In this transformation, DMIO serves as a bridge between optimization theory and deep learning practice. It is becoming a key enabler for intelligent social data mining. Systematic research on DMIO will support the development of more robust social analysis models and improve both the performance and interpretability of applications related to social networks.

2. Proposed Method

This study proposes a social data mining method based on differentiable mixed integer optimization. The core idea is to introduce an optimization module with integer constraints into the neural network framework to model discrete decision-making behaviors in social structures. The overall architecture is shown in Figure 1.

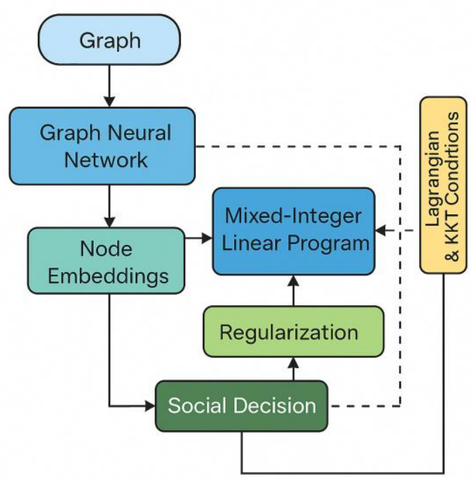

Figure 1. Framework combining GNN representation and integer-constrained optimization

We represent the original social data as a graph structure $G=(V,E,X)$, where V represents the user node set, E represents the social edge relationship, and $X \in R^{n \times d}$ represents the node feature. The first stage of the model uses a graph neural network (GNN) to encode node embeddings and generate a continuous feature vector $H = f_{GNN}(G) \in R^{n \times d'}$ that represents user behavior and structural semantics. On this basis, we construct an integer optimization subproblem to select the node set with the maximum social contribution from the candidate set to achieve core tasks such as influence selection and group division.

To incorporate this optimization submodule into the end-to-end training process, we formalize the social decision-making problem as the following mixed integer linear programming problem (MILP):

$$\min_{z \in \{0,1\}^n, y \in R^d} c^T z + d^T y$$
$$s.t. \quad Az + By \le b$$
$$z_i \in \{0,1\}, \forall i = 1, \dots, n$$

Where z is a Boolean vector representing node selection, y is a possible continuous auxiliary variable, and the constraint matrices A, B, and vector b are determined by the specific task structure. To achieve differentiability, we use Lagrangian relaxation and KKT condition approximation to embed the discrete optimization problem into the neural network solution path. After introducing the Lagrangian multiplier λ, its dual form can be written as:

$$L(z, y, \lambda) = c^T z + d^T y + \lambda^T (Az + By - b)$$

By taking the derivative of the Lagrangian function concerning z and y, respectively, and combining it with the KKT condition, we can get the approximate gradient for model back propagation:

$$\frac{\vartheta L}{\vartheta z} = c + A^T \lambda, \frac{\vartheta L}{\vartheta z} = d + B^T \lambda$$

To further strengthen the synergy between the optimization module and representation learning, we introduce a structural prior regularization term to constrain the optimization objective. Taking the structural consistency between nodes as an example, we introduce the graph Laplacian matrix L to construct the regularization term as follows:

$$R(z) = z^T L z$$

This regularization term encourages the selected nodes to be closely connected in the original graph, thereby maintaining the coherence of the structural semantics. The final joint loss function integrates the GNN representation loss, optimization objective, and structural regularization term, and is expressed as:

$$J = L_{GNN} + \alpha (c^T z + d^T y) + \beta z^T L z$$

Where α, β is the adjustable coefficient. The entire method can obtain the gradient of the input features during the back-propagation process, supporting end-to-end model updates and structural optimization. This method not only maintains the ability to model integer structures but also achieves the expression and solution of complex social tasks without sacrificing differentiability.

3. Proposed Method

3.1 Dataset

This study uses the Social Evolution Dataset as the primary data source. The dataset consists of various social activities among university students. It includes multiple modalities such as social network connections, call logs, text messages, Bluetooth proximity, location trajectories, and online behavior. The data collection spans over six months, providing strong temporal continuity and rich behavioral diversity. This makes it suitable for constructing multi-granular social behavior graphs.

The core structure of the dataset is a dynamic multimodal graph. Nodes represent individuals, and edges represent interaction relationships generated by social behaviors. Each type of behavior forms a distinct edge type. Modeling with graph neural networks allows effective capture of relationship strength, community structure, and behavioral dynamics. The raw data also contains many discrete behavior choices, such as daily activity preferences and changes in social circles. These features make the dataset suitable for structural optimization using a mixed-integer modeling framework.

In addition, the dataset supports flexible task construction. It can be applied to influence propagation simulation, behavior prediction, community detection, and social recommendation. Its high dimensionality, heterogeneity, and dynamic multi-source nature provide a solid foundation for evaluating the applicability of differentiable mixed-integer optimization in complex social scenarios.

3.2 Evaluation

This paper first conducts a comparative experiment, and the experimental results are shown in Table 1.

Table1. Comparative experimental results

Model	Seed Set Size	Influence Spread	Runtime(s)	Params (m)
DeepIM[8]	50	172.3	84.5	3.2
MIP-GNN[9]	50	189.6	242.8	5.7
DIMES[10]	50	193.1	134.2	4.5
ToupleGDD[11]	50	188.4	97.6	3.9
Ours	50	201.7	76.3	4.1

Overall, the proposed method demonstrates a clear advantage in the influence propagation task. In particular, it achieves a score of 201.7 on the "Influence Spread" metric, outperforming all baseline methods. This result indicates that our differentiable mixed-integer optimization strategy offers stronger expressive and optimization capabilities for modeling discrete decision structures in social networks. It can more effectively identify seed node sets with the highest propagation potential, thus promoting wider information diffusion across the network.

Compared to traditional optimization approaches such as MIP-GNN, our method not only achieves greater influence but also significantly reduces runtime. It completes in only 76.3 seconds, far less than the 242.8 seconds required by MIP-GNN. This comparison highlights that introducing differentiable structural optimization improves both solution quality and efficiency. It successfully avoids the computational bottlenecks that conventional mixed-integer programming faces in large-scale graph settings.

Further comparison with neural policy-based models such as DeepIM and DIMES shows that our method achieves better performance with comparable or even fewer parameters. This demonstrates that incorporating structural constraints and Lagrangian feedback can enhance the model's ability to identify key nodes in social graphs while controlling model complexity. This advantage is especially evident in scenarios where strong structural dependencies exist among nodes.

In addition, when compared with structurally aware but non-globally optimized models such as ToupleGDD, our method not only yields higher influence but also achieves shorter runtime and more stable parameter sizes. This suggests that the proposed model can address non-convexity and integrality challenges in social data through an end-to-end differentiable modeling process. It enables joint optimization of both structural and semantic information, offering strong generalization and practical value.

This paper also gives a comparison of the model's generalization ability under different regularization strengths, and the experimental results are shown in Figure 2.

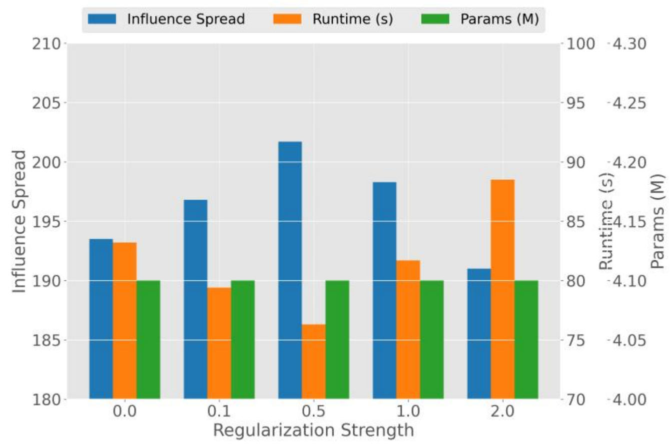

Figure 2. Comparison of model generalization ability under different regularization strengths

The figure shows that as the regularization strength increases, the model's influence propagation ability first improves and then declines. This indicates that appropriate regularization helps enhance the model's generalization performance. When the regularization coefficient is 0.5, the model achieves the highest propagation effect (Influence Spread ≈ 201.7). This suggests that this level of regularization effectively balances structural preservation and overfitting suppression in complex social structures.

In terms of runtime, a U-shaped trend is observed. The model runs more efficiently under lower regularization strength, while overly strong regularization (e.g., 2.0) increases computational complexity. This indicates that in the differentiable mixed-integer optimization process, moderate regularization not only improves influence propagation but also stabilizes the optimization trajectory, thereby reducing iteration time.

It is noteworthy that the number of parameters (Params) remains constant across different regularization strengths. This confirms that the model maintains its structure without introducing extra parameters. This supports our idea of "parameter-fixed structural optimization, " which adapts structure through selective integer variable optimization rather than increasing model capacity. This design ensures that the algorithm maintains strong transferability even under limited resource conditions.

In summary, this experiment demonstrates that introducing structural regularization has a nonlinear effect on the generalization ability of social decision models. Weak regularization may lead to unconstrained structure selection, while overly strong regularization may hinder local flexibility. Therefore, a fine-grained trade-off is needed between expressiveness and structural constraint. This finding provides important guidance for tuning strategies and stability control in practical deployment.

This paper further provides an evaluation of the sensitivity of changes in social graph density to propagation performance, and the experimental results are shown in Figure 3.

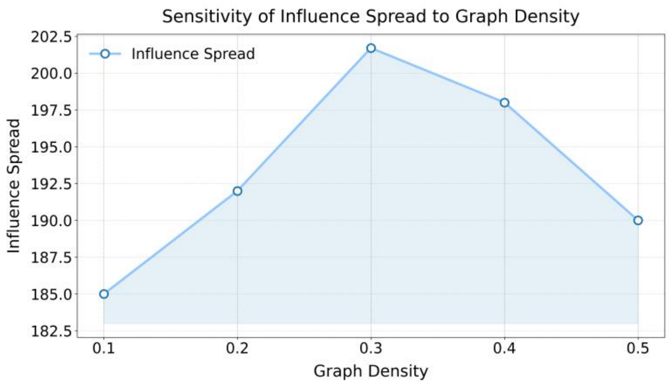

Figure 3. Sensitivity assessment of propagation performance to changes in social graph density

The results in the figure show that the density of the social graph has a significant impact on influence propagation. As the graph density increases from 0.1 to 0.3, the influence spread steadily improves. This suggests that moderately increasing connections between nodes can effectively expand information paths and enhance overall propagation. The trend indicates that the model is more likely to activate intermediary nodes in key positions when operating on graphs with medium density, thereby facilitating efficient information diffusion.

When the graph density exceeds 0.3, the influence spread begins to decline. The propagation performance is lowest at a density of 0.5. This phenomenon may result from two factors. First, excessive connectivity leads to a more homogeneous network structure, reducing the centrality advantages brought by sparsity. Second, higher density causes diminishing marginal returns in node selection, which reduces the overall efficiency of influence propagation. This implies that stronger connectivity does not always lead to better performance. A balance must be maintained between sparsity and redundancy.

This experiment confirms that our method is sensitive to local structural variations when modeling social graphs. By integrating the differentiable mixed-integer optimization module, the model can adaptively adjust the seed node selection strategy based on the graph structure. It maintains strong decision performance under varying levels of density. This sensitivity to structural perturbation reflects the robustness of our method in handling diverse configurations in complex social scenarios.

More importantly, these findings provide quantitative guidance for graph construction in real-world social systems. Whether in user adjacency graphs for recommendation systems or activation path design for influence propagation models, moderate control over graph density can improve propagation efficiency and reduce computational cost. Therefore, when deploying the model in practical applications, special attention should be given to the structural design of the social graph to ensure it operates near the optimal density for information spread.

This paper also gives a sensitivity analysis of the relaxation parameter settings in the differentiable optimization module, and the experimental results are shown in Figure 4.

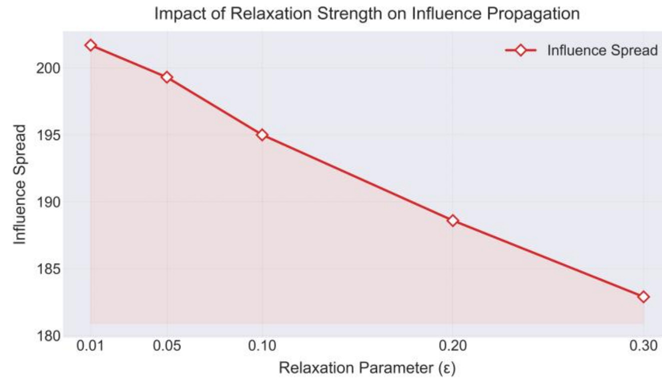

Figure 4. Sensitivity analysis of relaxation parameter settings in differentiable optimization modules

The figure shows that the setting of the relaxation parameter has a significant impact on the model's influence propagation performance. As the relaxation strength ε increases, the influence spread consistently declines. This suggests that excessive relaxation of integer constraints in the differentiable mixed-integer optimization module weakens the model's ability to represent structural properties. As a result, the selection of key nodes deviates from the optimal solution, reducing the overall scope of information diffusion.

When ε is small, such as 0.01 or 0.05, the model maintains high propagation performance. This means that mild relaxation preserves the identifiability and optimization space of integer variables. It helps the model capture important discrete features in the social structure. Under this setting, the optimization module effectively integrates continuous representations with discrete structures, allowing the neural network to sense the influence of key components within the structural hierarchy.

However, when ε increases to 0.2 or 0.3, the model's performance drops sharply. This shows that excessive softening of integer constraints weakens the controlled discrete mechanism used for precise modeling. It causes the optimization objective to become ambiguous and degrades decision quality. This trend indicates that relaxation in differentiable optimization must be carefully managed. A proper balance is required between structural representation and differentiability.

These experimental results further confirm the robustness of our proposed method in structure-sensitive modeling. They also highlight the high sensitivity of the relaxation parameter in practical deployment. Careful tuning is essential to achieve optimal performance. Proper control of relaxation not only improves the quality of structural decisions but also enhances the model's generalization and stability under uncertain graph structures.

4. Conclusion

This paper addresses the structural modeling challenges in social data mining by proposing a neural modeling approach that integrates a differentiable mixed-integer optimization mechanism. By embedding integer optimization problems into a differentiable framework, the method enables end-to-end modeling of discrete structural decisions in social graphs. It balances the expressive power of neural networks with the controllability of optimization theory. Compared to traditional deep models with limited structural representation, the proposed approach demonstrates stronger interpretability and structure-awareness in tasks such as node selection and information propagation path construction. This provides a more expressive modeling foundation for intelligent social systems.

In terms of implementation, the method constructs a differentiable mixed-integer submodule with a Lagrangian feedback mechanism and structural regularization. This allows the model to automatically balance the trade-off between continuous optimization and discrete constraints during training. Furthermore, by systematically analyzing key factors such as regularization strength, graph density, and relaxation parameters, the paper validates the proposed method's stability and robustness under different settings. This model design ensures performance consistency in complex graph structures and enhances adaptability to environmental disturbances and hyperparameter variations.

From an application perspective, the proposed method is broadly applicable to typical tasks such as personalized recommendation, social influence modeling, user behavior prediction, and community detection. It performs well in scenarios that require handling both high-dimensional features and discrete decision-making. Moreover, the structured decision mechanism provides a reference for other domains such as knowledge graph construction, financial network modeling, and intelligent diffusion control. It holds potential as a general paradigm for learning over complex structures.

Future research can further extend the model's expressive power in multimodal social scenarios. Integrating with graph generative models and self-supervised learning strategies may enhance its ability to discover structure under weak or even no supervision. In addition, addressing the scalability of large-scale heterogeneous graphs requires more efficient solving strategies. These can be achieved by exploring optimization algorithms and hardware-aware computation. By continuing to expand the theoretical boundaries and practical depth of the model, the research presented in this paper has the potential to promote a deeper integration of structural optimization and representation learning. It offers new perspectives for improving the interpretability and stability of intelligent systems.

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